

Metallicity and superconductivity of the hydrogen-rich compounds

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Technical University of Ostrava





Sources

<https://bitbucket.org/azja/qmt>

♠ J. Spałek et al., Phys. Rev. B **61**, 15676 (2000);

♣ APK et al., Eur. Phys. J. B **86**, 252 (2013);

♦ A. Biborski, APK, J. Spałek, Comput. Phys. Commun. **197**, 7 (2015);

♡ A. Biborski, APK, J. Spałek, Phys. Rev. B **98**, 085112 (2018).

1 Introduction

- Metalization of Hydrogen

2 Methods

- Electronic interactions
- Exact Diagonalization Ab Initio (EDABI++)

3 One-dimensional hydrogen

- Model
- State function
- Electronic properties
- Conclusions

4 Two-dimensional hydrogen

- Model
- Transition sequence
- Spin-ordering
- Metallicity
- Superconductivity
- Conclusions

ICH

The New York Times



Mount Etna, Europe's Most Active Volcano, Puts On a Show



In California, a Move to Ease the Pressures on Aging Dams



Edward E. David Jr., Who Elevated Science Under Nixon, Dies at 92



Before Vaquitas Vanish, a Desperate Bid to Save Them

SCIENCE

Hydrogen Squeezed Into a Metal, Possibly Solid, Harvard Physicists Say

By KENNETH CHANG JAN. 26, 2017



Wasserstoff zu Metall gequetscht?

VON WERNER LANGERHOF 16.01.2017 12:00:00

Ingeniøren

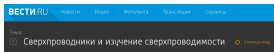
Myheder Blogs Debat Jobfinder Anviser Mere

VIDES PUNKT KUNSTIG INTELLIGENS 3D-PRINT DIESELSKANDALEN KAMPF FOR MILLI

Metallisk hydrogen sætter forskerverdenen i kog

Påstand om fremstilling af metallisk hydrogen mødes med meget hård kritik fra forskere. Lige til skraldepannen, lyder det. Andre bakker dog de kritiserede forskere op.

AF Jens Ramskov 2. feb 2017 kl. 12:03



Новости | Наука

36 news 007 15:08 1 Дарья Татарская

Прорыв в физике? Твёрдый металлический водород, возможно, стал реальностью



Scientific breakthrough lost? Unique

Andrzej P. Kądziaława

Le Scienze

RIVISTA DI SCIENZA E LETTERATURA

Le Scienze Metacorelle comportamento epidemiologia onde gravitazionali

21 gennaio 2017

Idrogeno solido metallico, un annuncio e molti dubbi



Credit: Piers F. D'Arco, F. Silvera

Due ricercatori hanno annunciato di aver prodotto per la prima volta idrogeno solido metallico, previsto per via teorica circa ottant'anni fa, un traguardo che aprirebbe la strada a nuove applicazioni, dai superconduttori ai propellenti per razzi. Ma non pochi scienziati nutrono dubbi riguardo alle modalità con cui è stato svolto l'esperimento e dunque al suo risultato (nol)

indiatoday NEWS

INDIA TODAY CONCLAVE 2017 ASSEMBLY ELECTIONS 2017 MAIL TODAY INDIA TODAY

World's first metallic hydrogen sample disappears

Last month physicists from Harvard University in the US had claimed to have successfully turned hydrogen into a metal - something researchers had been struggling to achieve for more than 80 years.

PTI | Posted by Bin Jose Boston, February 27, 2017 | UPDATED: 19:20 IST



Metallic Hydrogen

INDEPENDENT News, Politics, Culture, Lifestyle, Tech, Sport, Daily Edition, Classifieds

News

World's only piece of a metal that could revolutionise technology has disappeared, scientists reveal

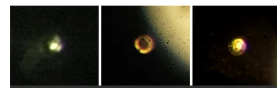
Scientists from Harvard University claim to have created the first ever piece of metallic hydrogen on Earth but have now lost it after creating it. Science of diamond building is under microscope



TECHNOLOGY NEWS | 27 JAN 2017 | 10:00 AM GMT

U.S. scientists create metallic hydrogen, a possible superconductor, ending quest

FULL COVERAGE INDIA ELECTIONS 2017



RMF 24 NAJBLIZEJ FAKTOW



RMF 24 | Polska | Nauka | Naukowcy twierdzą, że stworzyli metaliczny wodór, materiał o potencjalnie rewolucyjnych właściwościach. Na razie jego wytworzenie wymaga skrajnie niskiej temperatury i olbrzymiego ciśnienia, wiążącego, wół w ciemnym studziu (Zemci, żółci) i ciemności. W rzeczywistości, naukowcy, którzy twierdzą, że stworzyli metaliczny wodór, twierdzą, że stworzyli metaliczny wodór.

Metaliczny wodór, materiał marzeń, stał się rzeczywistością

Czwartek, 26 stycznia 2017

Jego istnienie fizycy przewidywali od 80 lat. Teraz zostało stać się faktem. Naukowcy z Uniwersytetu Harvarda ogłosili, że udało im się stworzyć metaliczny wodór, materiał o potencjalnie rewolucyjnych właściwościach. Na razie jego wytworzenie wymaga skrajnie niskiej temperatury i olbrzymiego ciśnienia, wiążącego, wół w ciemnym studziu (Zemci, żółci) i ciemności. W rzeczywistości, naukowcy, którzy twierdzą, że stworzyli metaliczny wodór, twierdzą, że stworzyli metaliczny wodór.

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Prague, Feb. 27, 2020

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Metalization of Hydrogen

Prediction: Metallic state

E. Wigner i H. B. Huntington, J. Chem. Phys. **3**, 764 (1935):

- $H - H$ distance (d_{HH}),
- Wigner-Seitz radius ($r_s \equiv (\frac{3}{4\pi n})^{1/3}$).

Metalization at $p \approx 25 \text{ GPa}$:

$$2r_s > d_{HH}.$$

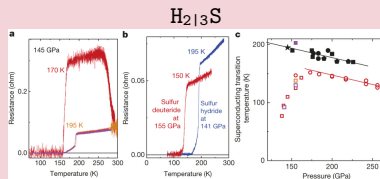
Prediction: Superconductivity in 300 K

N. Ashcroft, PRL **21**, 1748 (1968)

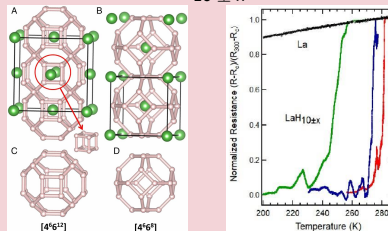
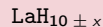
$$T_C = \Theta_D \mathcal{F}(\text{el.-ph.})$$

	$T_C \text{ (K)}$
Jupiter surface	$\sim 10^{-27}$
Jupiter core	~ 290

Hydrogen in 2D - superconductivity?



A. P. Drozdov et al., Nature **525**, 73 (2015)



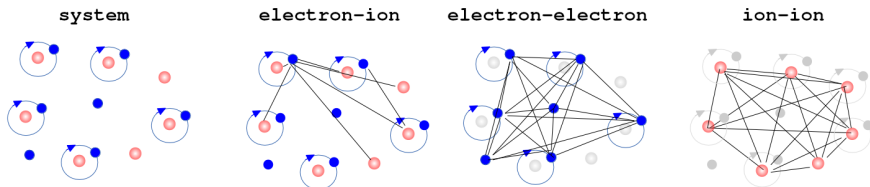
$L_{th.}$: Hanyu Liu et al., PNAS **114**, 27 (2017)

$R_{exp.}$: M. Somayazulu et al., arXiv:1808.07695 (2018)

Picture

Born–Oppenheimer Approximation

$$\psi^{\text{total}} = \psi^{\text{electrons}} \otimes \psi^{\text{nuclei}}$$



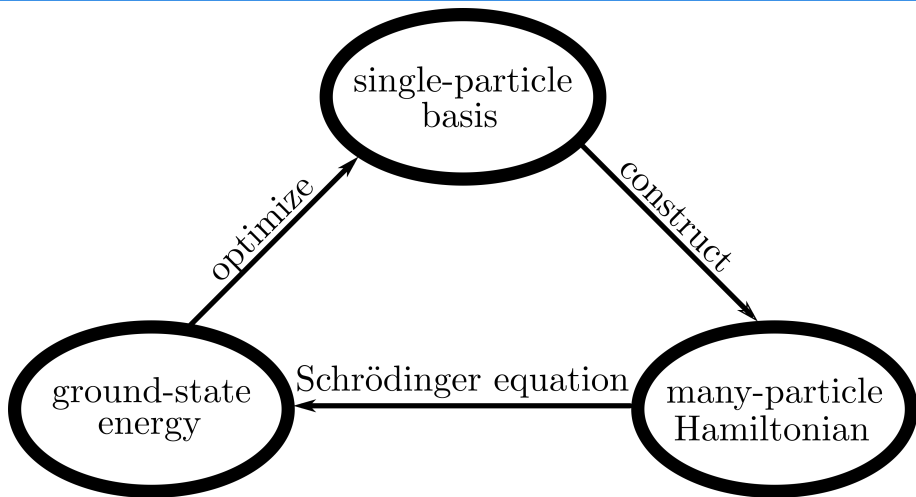
Hamiltonian

Rydberg atomic units $\hbar = 2m_e = \frac{e}{\sqrt{2}} = 1$

 $\mathcal{H} =$

$$-\sum_i \nabla_i^2 + \sum_{ij} \frac{\mathcal{V}_{\text{el.-ion}}}{|r_i - R_j|} + \sum_{i>j} \frac{\mathcal{V}_{\text{el.-el.}}}{|r_i - r_j|} + \sum_{i>j} \frac{\mathcal{V}_{\text{ion-ion}}}{|R_i - R_j|}$$

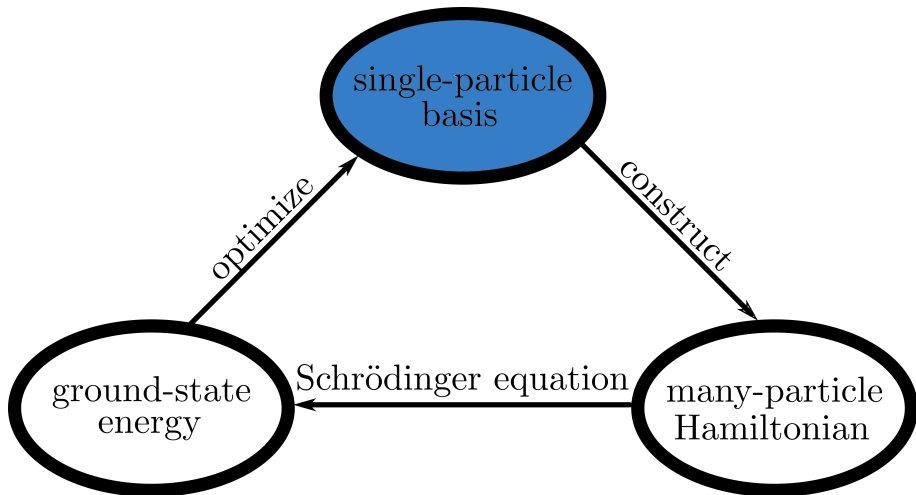
Exact Diagonalization Ab Initio (EDABI++)



Spiritus Movens: Using second quantization

Decoupling of single-particle picture and wavefunctions' algebra

Exact Diagonalization Ab Initio++ I



Single-particle basis

Conservation of complexity

Switching to the second quantization is effective only for the orthogonal bases (otherwise either $\{\hat{c}_i, \hat{c}_j^\dagger\} = \mathbb{S}_{ij}$ or $\hat{c}^i \equiv \mathbb{S}^{ij} \hat{c}_j^\dagger$).

LCAO

Orthogonal basis $\{w_i\}$ can be expressed as a linear combination of Slater orbitals $\{\psi_i\}$:

$$w_i(\mathbf{r}) = \sum_k \beta_{kj} \psi_j(\mathbf{r}),$$

satisfying orthonormality condition

$$\langle w_i | w_j \rangle = \delta_{ij}.$$

Suitable methods

Let us define mixing matrix $\mathbb{W}_{ij} \equiv \langle w_i | \psi_j \rangle$.

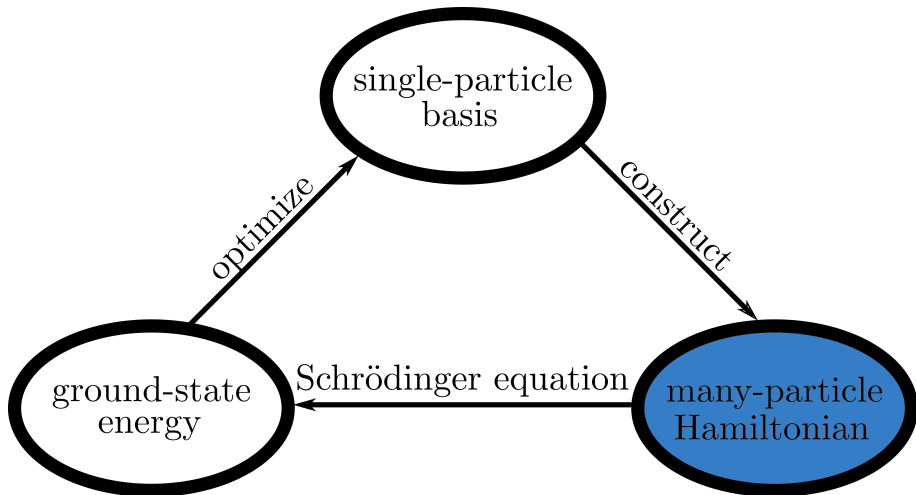
■ Löwdin orthogonalization

- ♪ one solution close to starting orbitals
- ⊗ dense $\mathbb{W}$
- ⊗ requires sharp cut-off for infinite systems

■ quadratic forms

- ⊗ many solutions
- ♪⊗ allows/requires symmetry constraints
- ♪ sparse $\mathbb{W}$
- ♪ systematic approach to infinity

Exact Diagonalization Ab Initio++ II



Hamiltonian

Hamiltonian and its parameters

By acting on the starting, first-quantization Hamiltonian with the field operator: $\Psi_i \equiv w_i(\mathbf{r})\chi_\sigma \hat{c}_{i\sigma}^\dagger$ we get the second-quantization Hamiltonian

$$\mathcal{H} = \sum_{\substack{i,j \\ \sigma}}^{\text{bound. con.}} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \sum_{\substack{i,j,k,l \\ \sigma,\sigma'}}^{\text{bound. con.}} V_{ijkl} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma'}^\dagger \hat{c}_{l\sigma'} \hat{c}_{k\sigma} + \mathcal{H}_{\text{ext}} + \mathcal{V}_{\text{ion-ion}}$$

with fermionic creation/annihilation operators

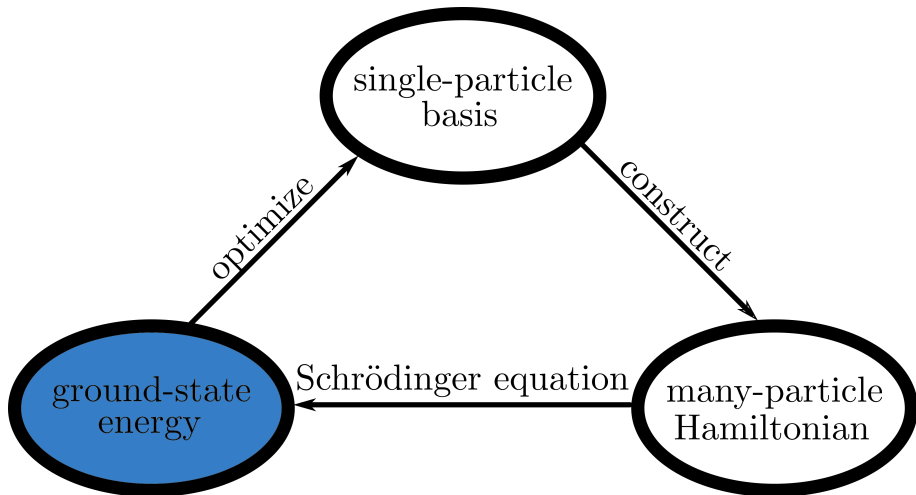
$$\{\hat{c}_{i\sigma}^\dagger, \hat{c}_{j\sigma'}^\dagger\} \equiv \{\hat{c}_{i\sigma}, \hat{c}_{j\sigma'}\} \equiv 0 \quad \text{and} \quad \{\hat{c}_{i\sigma}^\dagger, \hat{c}_{j\sigma'}\} \equiv \delta_{ij} \delta_{\sigma\sigma'}.$$

Hence, all of the information about single-particle wavefunction exists only in the microscopic parameters

$$t_{ij} = \left\langle w(\mathbf{r})_i \left| -\nabla^2 - \sum_{k=1}^n \frac{2}{|\mathbf{r} - \mathbf{R}_k|} \right| w(\mathbf{r})_j \right\rangle, \quad \text{"Rydberg" Atomic Units}$$

$$V_{ijkl} = \left\langle w(\mathbf{r})_i w(\mathbf{r}')_j \left| \frac{2}{|\mathbf{r} - \mathbf{r}'|} \right| w(\mathbf{r}')_k w(\mathbf{r})_l \right\rangle. \quad \hbar = 2m_e = \frac{e}{\sqrt{2}} = 1$$

Exact Diagonalization Ab Initio++ III



Diagonalization core

Advantage

No constraints on diagonalization method

Already applied:

- Lanczos
- Truncated Lanczos
- Variational Monte Carlo
- Gutzwiller Wavefunction
- Gutzwiller Approximation

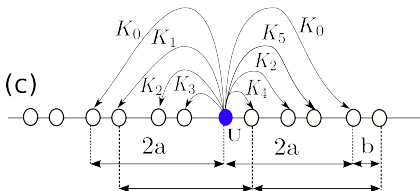
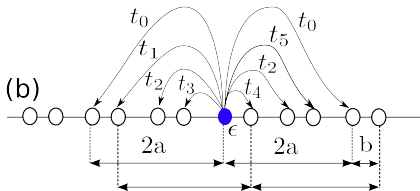
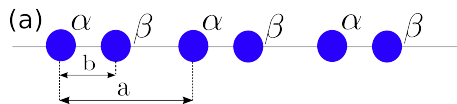
Whatever provides us with:

- ground-state energy (E_g)
- (ideally) excited states close to E_g
- (in)direct correlation picture
- observables for comparison with experiment
- scalability

2^{nd} -quantization states

$$|\Psi\rangle_N = \mathcal{N} \sum_k A_k |\Phi_k\rangle, \quad |\Phi_k\rangle = \prod_{i \in \Omega_{\uparrow k}} \hat{c}_{i\uparrow}^\dagger \prod_{j \in \Omega_{\downarrow k}} \hat{c}_{j\uparrow}^\dagger |0\rangle, \quad \langle \Phi_k | \Phi_l \rangle = \delta_{kl}$$

For details see: Phys. Rev. B **98**, 085112 (2018)



Assumptions

- (a) two hydrogen atoms in the unit cell (α, β), with the lattice parameter a and bond length b ;
- (b) range of the hoppings terms extends up to $2a$;
- (c) interactions counted up to the range of $2a$.

"Infinite" crystal

- Periodic Boundary Conditions;
- supercell of 17, 21, 25, 33 and 37 unit cells;

Hamiltonian

Second quantization

$$\begin{aligned} \mathcal{H} = & \sum_i \epsilon_i (\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}) + \sum_{i \neq j} t_{ij} (\hat{c}_{i\uparrow}^\dagger \hat{c}_{j\uparrow} + \hat{c}_{i\downarrow}^\dagger \hat{c}_{j\downarrow}) & // \text{ free electrons} \\ & + \sum_i U_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \sum_{i \neq j} K_{ij} \hat{n}_i \hat{n}_j & // \text{ interactions} \end{aligned}$$

First-to-second-quantization calculation step

$$\begin{aligned} t_{ij} &\equiv \left\langle w_i(\mathbf{r}) \left| -\nabla^2 - \sum_{l \in \text{ions}} \frac{2Z}{|\mathbf{R}_l - \mathbf{r}|} \right| w_j(\mathbf{r}) \right\rangle & \epsilon_i &\equiv t_{ii} \\ V_{ijkl} &\equiv \left\langle w_i(\mathbf{r}) w_j(\mathbf{r}') \left| \frac{2}{|\mathbf{r} - \mathbf{r}'|} \right| w_k(\mathbf{r}) w_l(\mathbf{r}') \right\rangle & U_i &\equiv V_{iiii}, \quad K_{ij} \equiv V_{ijij} \end{aligned}$$

Dimensionality - 1D chain in 3D space

- $w_i(\mathbf{r})$ build from 1s Slater orbitals;
- Coulomb potential $V_C(\mathbf{R}) \propto |\mathbf{R}|^{-1}$;

Proper state function

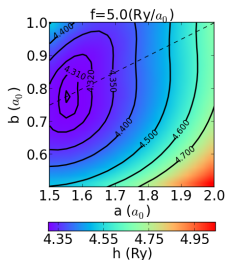
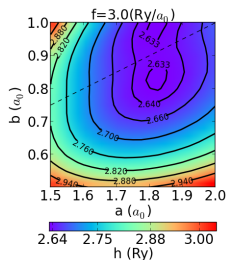
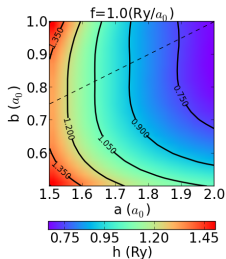
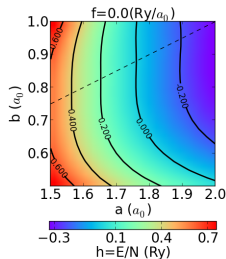
One-dimensional enthalpy

$$h \equiv f \frac{a}{2} + \frac{E}{N},$$

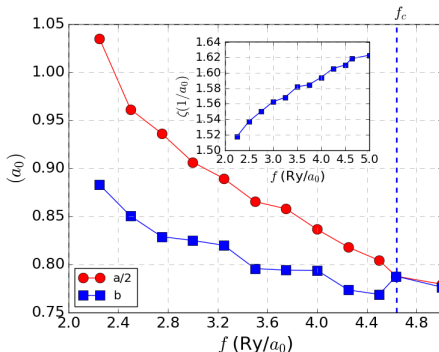
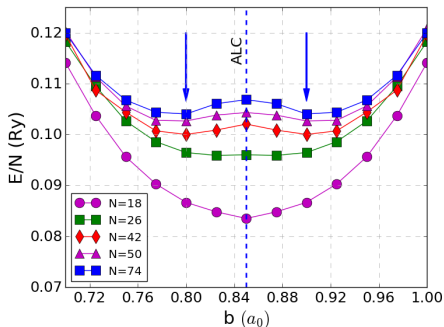
with f as an external force
(analogue of the pressure),
the lattice parameter a ,
and ground-state energy E for
the N -particle supercell.

Run for given f

- opt. structure
- opt. wavefunction
- opt. Jastrow



Results for finite systems

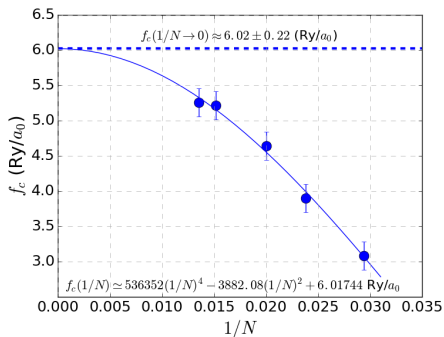
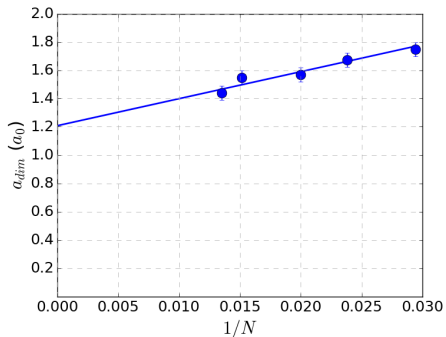


Peierls-like distortion from First Principles for a correlated system

- No distortion for small systems.
- Molecular $\rightarrow$ atomic transition at high “pressure”
 $\rightarrow$ reverse Peierls-like transition.

for finite systems cf. also E. Giner *et al.*, J. Chem. Phys. **138**, 074315 (2013).

Thermodynamic limit

Conditions of molecular-to-atomic transition for $N \rightarrow \infty$

- finite-size scaling of atomization lattice parameter $a_{dim} \approx 1.17a_0 > 0$;
- finite-size scaling of atomization force $f_c \approx 6.02 \frac{Ry}{a_0} < \infty$.

Metallicity of hydrogen chain

Point of reference

We use the equilibrium microscopic parameters of the Hamiltonian for $N = 50$ as a point of reference.

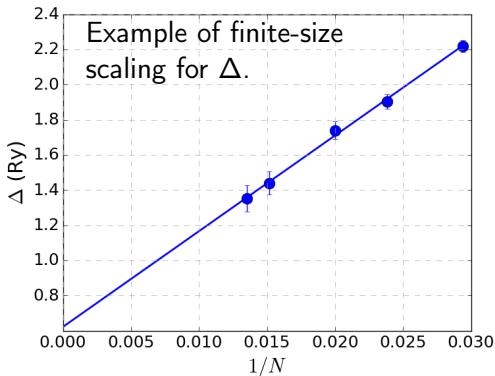
Charge gap

$$\Delta_N \equiv \frac{E_{N+4} - 2E_N + E_{N-4}}{4} \Big|_{@h(f)}$$

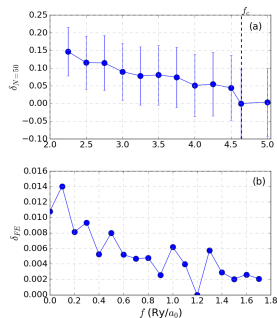
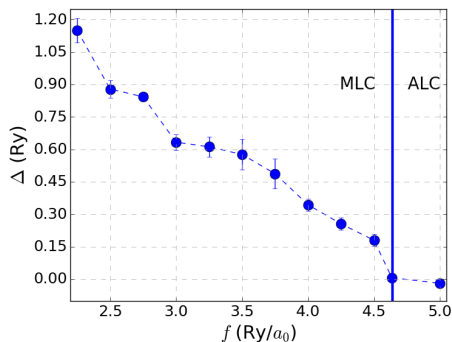
E_N - the ground state of the N -particle system described by the reference Hamiltonian with the structure minimizing the effective enthalpy.

Thermodynamic limit

$$\Delta \equiv \Delta_\infty = \lim_{N \rightarrow \infty} \Delta_N$$



Closing of the charge gap



Apparent metallicity of the hydrogen chain in the atomic phase

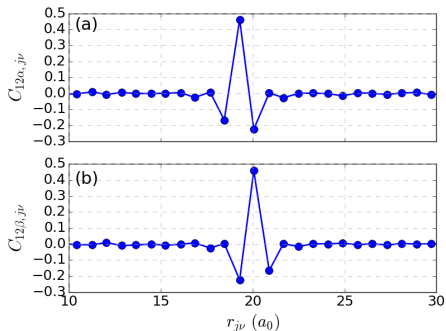
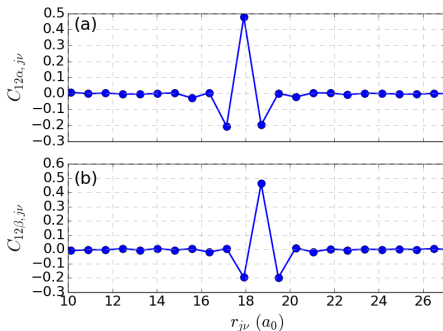
- charge gap closed at the MLC $\rightarrow$ ALC transition;
- further-than-nearest neighbor hoppings;
- chain exist in 3D (both single-particle wavefunctions and Coulomb potential are taken for $D = 3$);

in agreement with L. Stella *et al.*, Phys. Rev. B **84**, 245117 (2011)

Density-density correlation

Density-density correlation

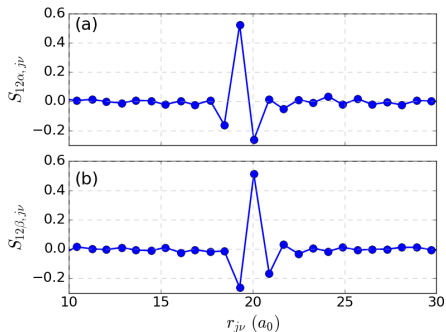
$$C_{i,j} \equiv \langle \hat{n}_i \hat{n}_j \rangle - \langle \hat{n}_i \rangle \langle \hat{n}_j \rangle$$

(LEFT) $f = 4.5 \text{ Ry}/a_0$;(RIGHT) $f = 5.0 \text{ Ry}/a_0$

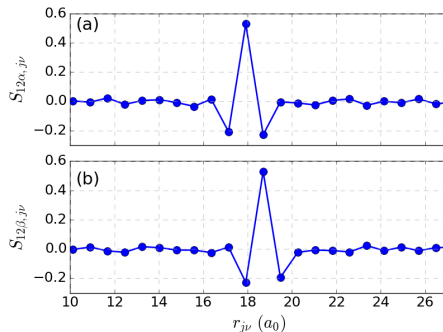
Spin-spin correlation

Spin-spin correlation

$$S_{i,j} \equiv \langle (\hat{n}_{i\uparrow} - \hat{n}_{i\sigma})(\hat{n}_{j\uparrow} - \hat{n}_{j\sigma}) \rangle = \langle \hat{S}_i^z \hat{S}_j^z \rangle$$



(LEFT) $f = 4.5 \text{ Ry}/a_0$;

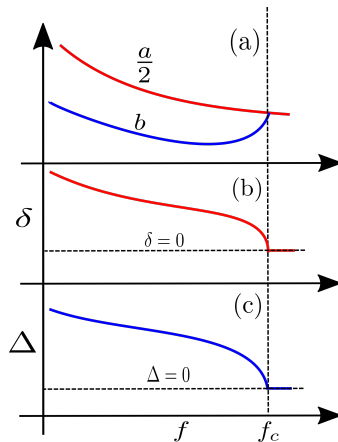


(RIGHT) $f = 5.0 \text{ Ry}/a_0$

Conclusions 1D

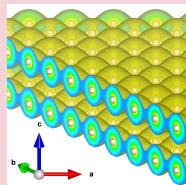
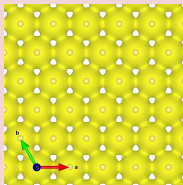
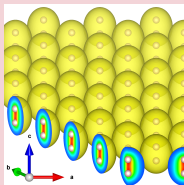
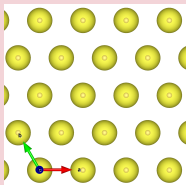
Hydrogen chain

- Peierls-like distortion at ambient “pressure”;
- correlations do not weaken distortion;
- external force induces molecular $\rightarrow$ atomic transition;
- concomitant atomization and metallization ;
- no long-range order;



Triangular lattice

Two-dimensional crystal



- periodic boundary conditions in xy plane;
- Lanczos algorithm for the diagonalization core of 6 and 8 atoms (to comply with proper Néel 120° and 90° phases);

- wavefunction constructed from 10 classes of nodes

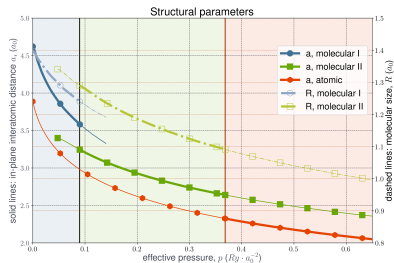
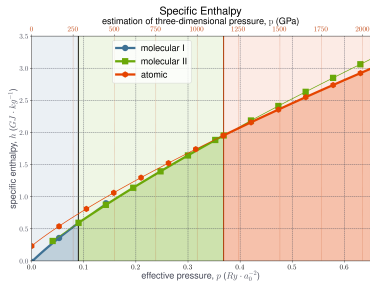
$$\mathcal{H} = \sum_{i\sigma} \epsilon_i \hat{n}_{i\sigma} + \sum_{i \neq j \sigma} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} \quad \hookrightarrow \text{hoppings } t_{ij} \text{ up to } 10^{\text{th}} \text{ neighbor;}$$

$$+ \sum_i U_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \sum_{i \neq j} K_{ij} \hat{n}_i \hat{n}_j \quad \hookrightarrow \text{Coulomb repulsion } K_{ij} \text{ up to } 10^{\text{th}} \text{ neighbor;}$$

$$- \sum_{i \neq j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \sum_{i \neq j} J_{ij} \hat{n}_i \hat{n}_j \quad \hookrightarrow \text{ferromagnetic exchange } J_{ij}$$

$$+ \sum_{i \neq j} J_{ij} \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\downarrow}^\dagger \hat{c}_{j\downarrow} \hat{c}_{j\uparrow} \quad \text{up to } 3^{\text{rd}} \text{ neighbor;}$$

2D enthalpy and lattice parameters

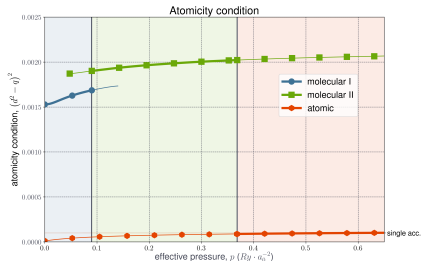


Question:

What is the quantum equivalent of $R_{\text{eff}} \rightarrow \infty$?

$$\delta d \equiv \left(P \begin{pmatrix} * \\ \uparrow\downarrow \end{pmatrix} P \begin{pmatrix} \uparrow\downarrow \\ * \end{pmatrix} - P \begin{pmatrix} \uparrow\downarrow \\ \uparrow\downarrow \end{pmatrix} \right)^2$$

$$\equiv \left(\langle \Phi_0 | \hat{n}_{1\uparrow} \hat{n}_{1\downarrow} | \Phi_0 \rangle \langle \Phi_0 | \hat{n}_{2\uparrow} \hat{n}_{2\downarrow} | \Phi_0 \rangle - \langle \Phi_0 | \hat{n}_{1\uparrow} \hat{n}_{1\downarrow} \hat{n}_{2\uparrow} \hat{n}_{2\downarrow} | \Phi_0 \rangle \right)^2$$



Magnetic order

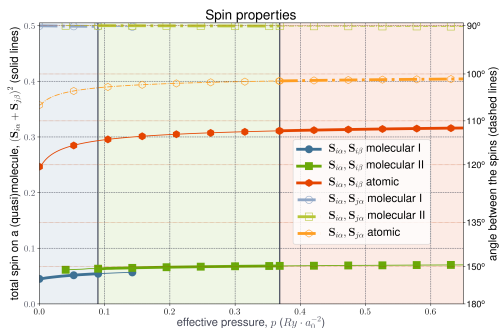
FM vs. AFM exchange

J_{FM} , Hund-like $\ll J_{\text{AFM}}$, kinetic

Required for the ambient pressure stability of the atomic phase!

Spin correlation

- 1 Molecular phases:
molecular near spin-singlet H_2
- 2 Atomic phase:
near 120° Néel order



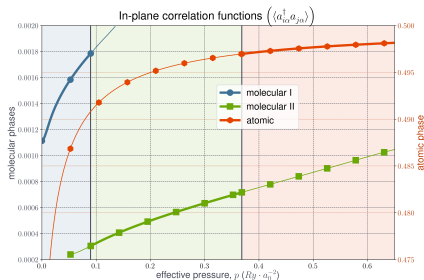
Total spin

	mol. I $\rightarrow$ II		mol. II $\rightarrow$ atomic	
$\ S\ _{\text{molecule}}$	0.10	0.14	0.16	0.54
$\ S\ _{\text{triangle}}$	0.86	0.87	0.86	0.077

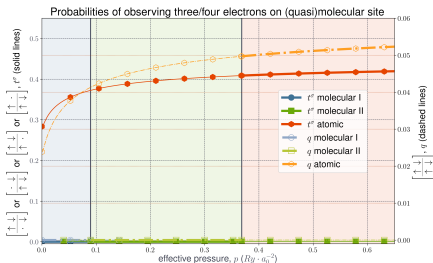
$$\|S\|_{\text{molecule}} \equiv \|S(x_{2D}, -\frac{R}{2}) + S_2(x_{2D}, \frac{R}{2})\|$$

$$\|S\|_{\text{triangle}} \equiv \|S(x_{2D}, \frac{R}{2}) + S(x_{2D} + e_1, \frac{R}{2}) + S(x_{2D} + e_2, \frac{R}{2})\|$$

Metallization I: Correlation Functions



$$C_{ij} \equiv \langle \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma} \rangle = \langle \Phi_0 | \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma} | \Phi_0 \rangle_G$$

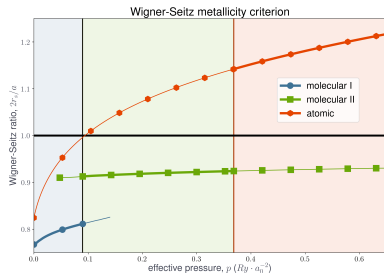


$$q \equiv P \begin{pmatrix} \uparrow\downarrow \\ \uparrow\downarrow \end{pmatrix} \quad d_0 \equiv P \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}$$

$$t_\uparrow \equiv P \begin{pmatrix} \uparrow \\ \uparrow\downarrow \end{pmatrix} \quad d_\uparrow \equiv P \begin{pmatrix} \uparrow \\ \uparrow \end{pmatrix}$$

$$t_\downarrow \equiv P \begin{pmatrix} \downarrow \\ \uparrow\downarrow \end{pmatrix} \quad d_\downarrow \equiv P \begin{pmatrix} \downarrow \\ \downarrow \end{pmatrix}$$

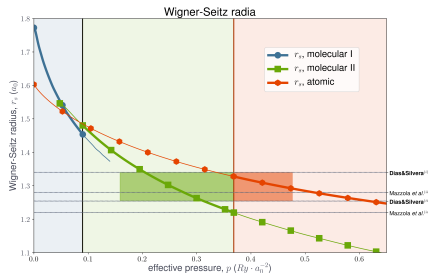
Metallization II: Wigner-Seitz Criterion



$$r_s \equiv \left(\frac{3}{4\pi n}\right)^{1/3}$$

$$\text{metal} \Leftrightarrow 2r_s > d_{HH}$$

Can be found experimentally!



source	method	$r_s(a_0)$
Min et al., PRB 33 , 324 (1986)	LMTO	2.85
Pfrommer et al., PRB 58 , 12680 (1998)	GGA-PW91	2.50
Svane et al., SSC 76 , 851 (1990)	LSDA	2.45
Li et al. PRB 66 , 035102 (2002)	LSDA	2.78
Li et al. PRB 66 , 035102 (2002)	PBE	2.50
Mazzola et al., Nat.C. 5 , 3487 (2014) ⁽ⁱ⁾	DMC + MD	1.28⁽ⁱⁱ⁾
McMinis et al., arXiv:1309.7051 (2013)	DMC	2.27
AB,APK,JS, PRB 96 , 085101 (2017) ⁽ⁱⁱⁱ⁾	EDABI	1.27
<i>molecular II</i>		EDABI 1.22 ^{+0.17} _{-0.06}
<i>atomic</i>		EDABI 1.33 ^{+0.10} _{-0.04}
Dias & Silveira		experiment 1.297(43)

Metallization III: Band structure

Bare bands

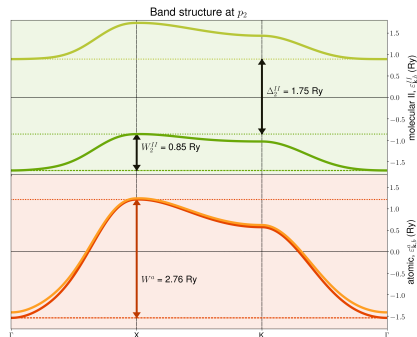
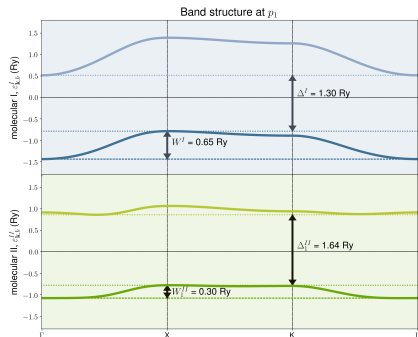
- easily calculable
- depend only on $\mathcal{H}_{\text{free}}$

Correlated bands

- full $\mathcal{H}$ dependence
- no generic method

Bands + Correlator

- calculable
- ⊗ correlator physics



Possibility of superconducting state

Conventional Superconductivity

Atomic hydrogen is **metallic** $\Leftrightarrow$ **McMillan formula** for critical temperature

McMillan formula

$$T_C = \frac{\Theta_D}{1.45} \exp \left[-\frac{1.04(1 + \lambda)}{\lambda + \mu^*(1 + 0.62\lambda)} \right]$$

- Θ_D - Debye temperature (from phonon DOS)
- λ - electron phonon coupling (from phononic and electronic dispersions)
- μ^* - Morel-Anderson pseudopotential - typically fitted to experimental data

We attempt to derive the ab-initio value of pseudopotential μ^* .

Morel-Anderson pseudopotential

$$\mu^* = \frac{\mu}{1 + \mu \log\left(\frac{T_{\text{phonons}}}{T_{\text{electrons}}}\right)}$$

$$\mu^* = \frac{n(E_F)(U - K_1)}{1 + n(E_F)(U - K_1) \log\left(\frac{E_F}{k_B \Theta_D}\right)}$$

Electron - phonon coupling

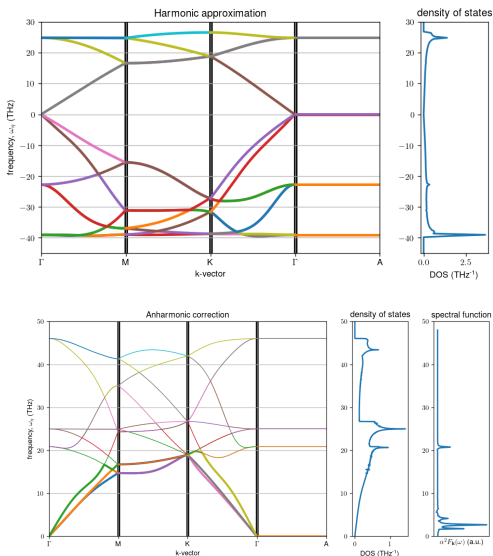
Eliashberg spectral function

$$\alpha^2 F_{\mathbf{k}}(\omega) \sim \sum_{\eta} \int d\mathbf{q} M_{\eta}^2 \delta(\omega - \omega_{\eta}) \delta(\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k} + \mathbf{q}))$$

allows us to obtain electron-phonon coupling constant

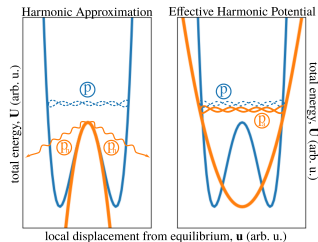
$$\lambda = 2 \int_0^{\infty} \frac{d\omega}{\omega} \alpha^2 F_{\mathbf{k}_f}(\omega)$$

Electrons and Phonons: DFT calculations with EDABI constraints



We take the Mexican-hat potential:

$$U(\{u^i\}) = U_0 + \frac{1}{2}\Phi_{ij}u^{ij} + \frac{1}{4!}\Phi_{ijkl}u^{ijkl}$$



$$F_i \rightarrow F_i + \frac{1}{4!}\Phi_{ij\langle kl\rangle}u^{ij\langle kl\rangle}.$$

At $p_{\text{eff}} = 0.7 R y a_0^{-2}$ ($\sim 1 \text{TPa}$)

$U_{\text{eff}} \equiv U - K_{pl} \text{ (Ry)}$	μ^*	λ
1.194	0.192	1.05
$\Theta_D \text{ (K)}$	$T_C \text{ (K)}$	$T_{AD} \text{ (K)}$
1300	164	176

DFT (VASP): SCAN meta-GGA + vdW

corrections + charge from EDABI

Conclusions 2D

Physics of hydrogen planes

- concomitant atomization & metallization;
- long-range interactions ($\sim ||R||^{-P}$);
- London-like interactions in insulating molecular phases (true molecular crystal);
- weak London-like attraction of atomic planes;
- benchmark for infinite-system quantum chemistry

(EDABI +  QMT);

Hydrogen-induced superconductivity

- medianly correlated system;
- anharmonic correction to force constants necessary;
- superconductivity induced by electron-phonon coupling;
- Morel-Anderson pseudopotential from First Principles;
- high critical temperature $T_C = 176K$;
- extreme pressure (chemical?);

Thank you for your attention

