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Rigorous analysis of many-electron effects in nanosystems: Quantum dot - ring nanostructure

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MOTIVATION

Quantum dot (QD) – nanoring (QR) systems (DRN) exhibit an interesting set of features. Namely, there are theoretical predictions, that such a structure will be susceptible to the so-called *wave-function engineering*, allowing to control the transport properties of the system, as well as the optical absorption, with the quantum dot potential depth [1,2]. The natural consequence if to study system's features for $N_e > 1$ electrons.

Our aim is to

- explicitly calculate the Coulomb interaction terms;
- calculate the many-electron states;
- describe the energy spectrum and eventual degeneracy of ground- and first-excited-states;
- calculate the probability of finding an electron in quantum dot and nanoring.

METHOD

We start the many-electron description with defining the field operators

$$\hat{\Psi}_\sigma(\mathbf{r}) = \sum_{i=1;\sigma=\pm1}^{M} \varphi_{i\sigma}(\mathbf{r})\hat{c}_{i\sigma}; \quad \hat{\Psi}^\dagger_\sigma(\mathbf{r}) \equiv \left(\hat{\Psi}_\sigma(\mathbf{r})\right)^\dagger,$$

where $\{\varphi_{i\sigma}\}$ is an orthonormal basis of single-particle wavefunctions, and $\hat{c}_{i\sigma}$ ($\hat{c}^\dagger_{i\sigma}$) are annihilation (creation) operators of electron with spin σ on i orbital.

The Hamiltonian now reads

$$\begin{aligned}\mathcal{H} \equiv & \sum_\sigma \int d^3r \hat{\Psi}^\dagger_\sigma(\mathbf{r})\mathcal{H}_1\hat{\Psi}_\sigma(\mathbf{r}) \\ & + \frac{1}{2}\sum_{\sigma\sigma'}\iint d^3r d^3r' \hat{\Psi}^\dagger_\sigma(\mathbf{r})\hat{\Psi}^\dagger_{\sigma'}(\mathbf{r}')V(\mathbf{r}-\mathbf{r}')\hat{\Psi}_{\sigma'}(\mathbf{r}')\hat{\Psi}_\sigma(\mathbf{r}) \\ = & \sum_{ij}\sum_\sigma t_{ij}\hat{c}^\dagger_{i\sigma}\hat{c}_{j\sigma} + \frac{1}{2}\sum_{ijkl}\sum_{\sigma,\sigma'} V_{ijkl}\hat{c}^\dagger_{i\sigma}\hat{c}^\dagger_{j\sigma'}\hat{c}_{l\sigma'}\hat{c}_{k\sigma},\end{aligned}$$

where

$$t_{ij} = \int d^3r \varphi^*_i(\mathbf{r})\mathcal{H}_1\varphi_j(\mathbf{r}),$$

$$V_{ijkl} = \iint d^3r d^3r' \varphi^*_i(\mathbf{r})\varphi^*_j(\mathbf{r}')\frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}'|}\varphi_l(\mathbf{r}')\varphi_k(\mathbf{r}).$$

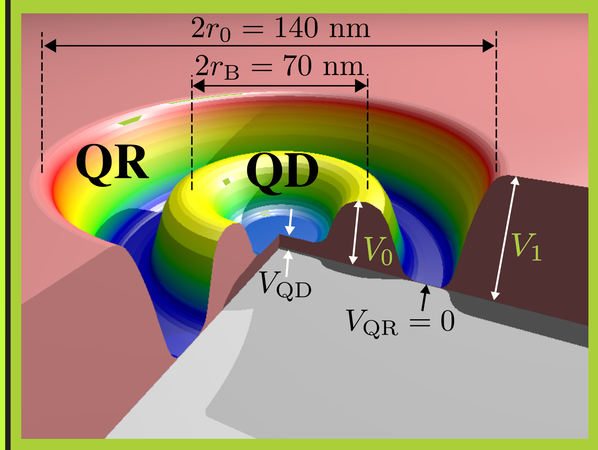
Hamiltonian is diagonalized for set, *large enough*, finite basis of single-particle wavefunctions, setting microscopic parameters t_{ij}, V_{ijkl} (see *Microscopic parameters*).

As we selected the single-particle basis (see *Single-particle basis*) to be the lowest-energy eigenstates of single-particle Hamiltonian, a condition holds

$$t_{ij} = \epsilon_i\delta_{ij},$$

where ϵ_i is a single-particle energy, and δ_{ij} Kronecker delta.

SINGLE-PARTICLE PROBLEM



Quantum dot - ring nanostructure - shape of the potential

Solution of single-particle problem

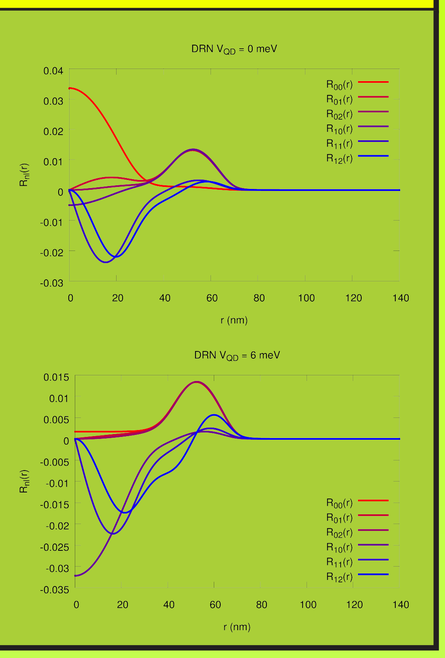
$$\left(\frac{\mathbf{p}^2}{2m^*} + V(\mathbf{r})\right)\psi_{nl}(\mathbf{r}) = \epsilon_{nl}\psi_{nl}(\mathbf{r}),$$

with potential $V(\mathbf{r})$ as shown on the left side, are the wavefunctions [1,2]

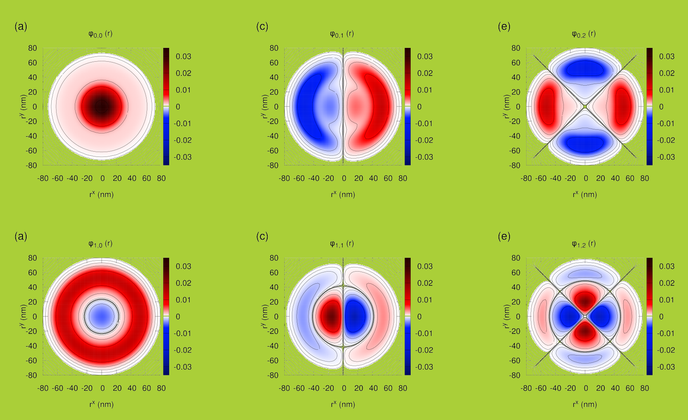
$$\psi_{nl}(\mathbf{r}) = R_{nl}(r)exp(il\phi).$$

For the degenerate case $\epsilon_{nl} = \epsilon_{n\bar{l}}$ and $R_{nl}(r) = R_{n\bar{l}}(r)$ we can write the real solutions

$$\varphi_{nl}(\mathbf{r}) = \frac{\psi_{n|l|}(\mathbf{r}) + sgn(l)\psi_{n|\bar{l}|}(\mathbf{r})}{\sqrt{2sgn(l)}}.$$



SINGLE-PARTICLE BASIS



Selected singe-particle wavefunctions in real basis $\{\varphi_i\}$ for quantum-dot potential depth $V_{\mathrm{QD}} = 0meV$.

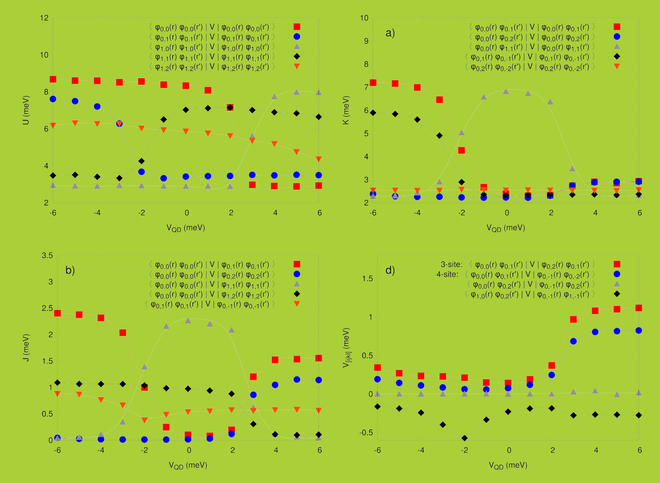
QUANTUM METALLIZATION TOOLS

QMT Quantum Metallization Tools

https://bitbucket.org/azja/qmt

Calculation where carried out using the QMT library, suitable for similar problem solutions [3].

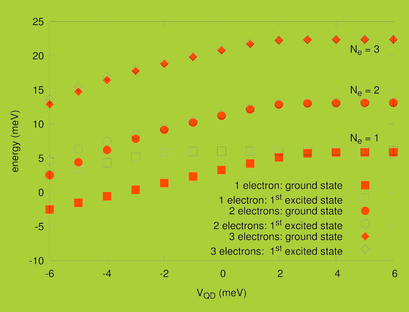
MICROSCOPIC PARAMETERS



Evolution of selected microscopic parameters: Hubbard on-site repulsion $U_i \equiv V_{iiii}$, interstate repulsion $K_{ij} \equiv V_{ijij}$, exchange integral $J_{ij} \equiv V_{ijji}$, and multistate parameters $V_{[ijkl]}$.

All integrals (with $M = 10$ single-particle wavefunctions - 10000) are calculated by means of Monte Carlo method with CUBA library [4], with accuracy of 0.005 meV. Rapid changes of parameters value around $V_{\mathrm{QD}} = -2meV$ and $V_{\mathrm{QD}} = 3meV$ are correlated to energy level crossing and repulsion.

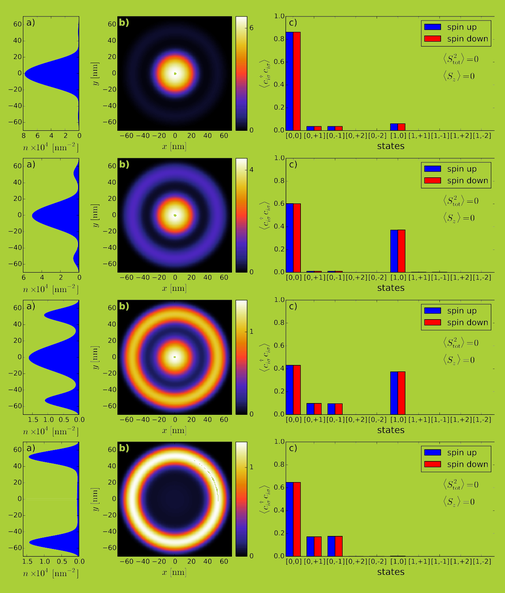
RESULTS: 2 I 3 ELECTRONS



Ground- and first-excited-state energy for $N_e = 1, 2, 3$.

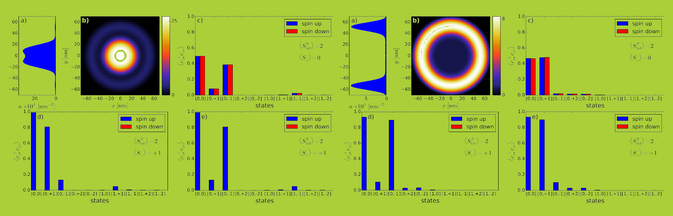
2 electrons

We observe evolution of many-particle state from the dot-oriented electrons for low values of V_{QD}, to the ring-oriented electrons for larger values of V_{QD}.



Density profiles (a), b)) and state occupancies (c)) for $V_{\mathrm{QD}} = -4meV, -2meV, 2meV, 4meV$

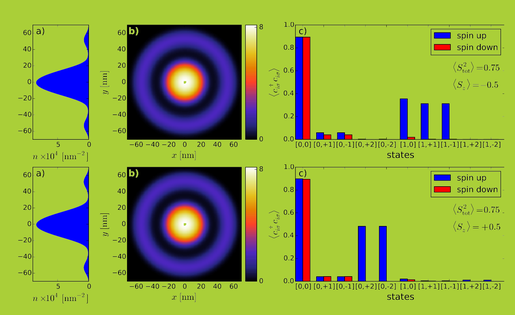
Similar behavior is present for the first-excited state.



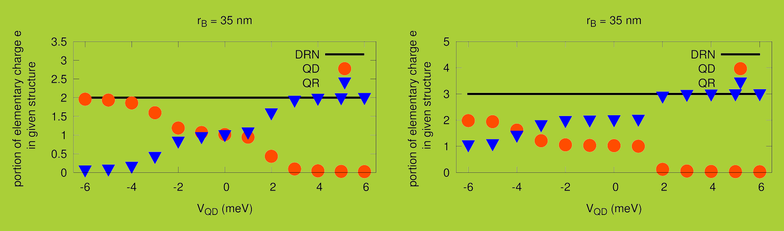
Density profiles (a), b)) and state occupancies (c)) for $V_{\mathrm{QD}} = -6meV, 4meV$

3 electrons

For three electrons we observe evolution from mixed state, where two electrons are in dot, whereas third one in the ring, to the ring-oriented electrons.



Density profiles (a), b)) and state occupancies (c)) for $V_{\mathrm{QD}} = -6meV$



Portion of elementary charge in quantum dot (QD) and nanoring (QR) versus dot potential V_{QD} for 2 (L) i 3 (R) electrons.

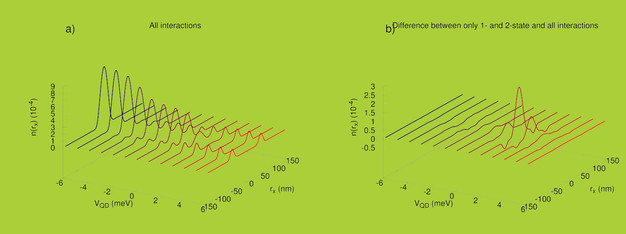
DEGENERACY

Degrees of degeneracy for different QD potential, for $N_e = 2, 3$.

	2 electrons				3 electrons			
	ground state		first-excited state		ground state		first-excited state	
V_{QD} (meV)	deg.	S_{tot}	deg.	S_{tot}	deg.	S_{tot}	deg.	S_{tot}
-6	1	0	3×2	1	2×3	1/2	2×2	1/2
-5	1	0	3×2	1	2×3	1/2	2×2	1/2
-4	1	0	3	1	2×3	1/2	4×2	3/2
-3	1	0	3	1	2×3	1/2	4×2	3/2
-2	1	0	3	1	2×3	1/2	4×2	3/2
-1	1	0	3	1	2×3	1/2	4×2	3/2
0	1	0	3	1	2×3	1/2	4×2	3/2
1	1	0	3	1	2×3	1/2	4×2	3/2
2	1	0	3×2	1	4	3/2	2×2	1/2
3	1	0	3×2	1	4	3/2	2×2	1/2
4	1	0	3×2	1	4	3/2	2×2	1/2
5	1	0	3×2	1	4	3/2	2×2	1/2
6	1	0	3×2	1	4	3/2	2×2	1/2

INFLUENCE OF MANY-STATE PARAMETERS

In presented method we are able to turn on and off selected interaction parameters to study their importance.



Evolution of electron density profile versus the QD potential V_{QD} (L) and the difference in density profiles for the three- and four-state interaction parameters turned off (R).

ACKNOWLEDGMENTS

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