

# First-principle approach to correlated realistic molecular hydrogen planes: Role of the Heisenberg-type interaction and the superconductivity

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# Outline

- 1 Motivation
  - Media frenzy
  - Hydrogen under pressure
- 2 Methods
  - EDABI + VMC
  - Model
- 3 Results
  - Transition sequence
  - Metallicity
  - Superconductivity
- 4 Conclusions

# R. P. Dias, I. F. Silvera, Science 10.1126/science.aa11579 (2017)



SCIENCE

## Hydrogen Squeezed Into a Metal, Possibly Solid, Harvard Physicists Say

By KENNETH CHANG JAN. 26, 2017



Frankfurter Allgemeine  
Wissen

Wasserstoff zu Metall gequetscht?  
VORBEREITUNG LINDBERG - ILLUSTRATION: J. L. P. / ILLUSTRATION: J. L. P.

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## Metallisk hydrogen sætter forskerverdenen i kog

Påstand om fremstilling af metallisk hydrogen mødes med meget hård kritik fra forskere. Lige til skraldependen, lyder det. Andre bakker dog de kritiserede forskere op.

af Jens Rasmussen 2. feb 2017 kl. 12:03

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## Le Scienze

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02 gennaio 2017

### Idrogeno solido metallico, un annuncio e molti dubbi

Due ricercatori hanno annunciato di aver prodotto per la prima volta idrogeno solido metallico, previsto per via teorica circa vent'anni fa, un traguardo che apre la strada a nuove applicazioni, dai superconduttori ai propellenti per razzi. Ma non pochi scienziati nutrono dubbi riguardo alle modalità con cui è stato svolto l'esperimento e dunque al suo risultato (1/17)

CONF STATE? Daniele F. Sessa

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### World's first metallic hydrogen sample disappears

Last month physicists from Harvard University in the US had claimed to have successfully turned hydrogen into a metal - something researchers had been struggling to achieve for more than 80 years.

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### World's only piece of a metal that could revolutionise technology has disappeared, scientists reveal

Scientists say a tiny amount of metallic hydrogen, long sought as a key to new superconductors, has disappeared after researchers at Harvard University announced its creation.

REUTERS

TECHNOLOGY NEWS | THE NEW YORK TIMES | SCIENTIST

### U.S. scientists create metallic hydrogen, a possible superconductor, ending quest

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FOX NEWS Tech

SCIENTIFIC BREAKTHROUGH LOST? UNIQUE METALLIC HYDROGEN SAMPLE DISAPPEARS

# Hydrogen under pressure

## TH: Metallic state (?)

E. Wigner i H. B. Huntington,  
J. Chem. Phys. **3**, 764 (1935):

- $H - H$  distance ( $d_{HH}$ ),
- Wigner-Seitz radius ( $r_s \equiv (\frac{3}{4\pi n})^{1/3}$ ).

Metalization at  $p \approx 25 \text{ GPa}$ :  
 $2r_s > d_{HH}$ .

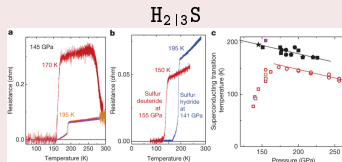
## TH: Superconductivity in 300K (?)

N. Ashcroft, PRL **21**, 1748 (1968)

$$T_C = \Theta_D \mathcal{F}(\text{el.-ph.})$$

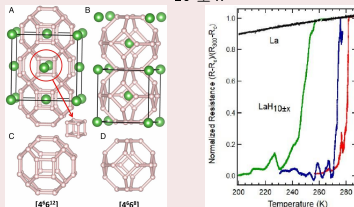
	$T_C$ (K)
Jupiter surface	$\sim 10^{-27}$
Jupiter core	$\sim 290$

## Hydrogen in 2D - superconductivity?



A. P. Drozdov et al., Nature **525**, 73 (2015)

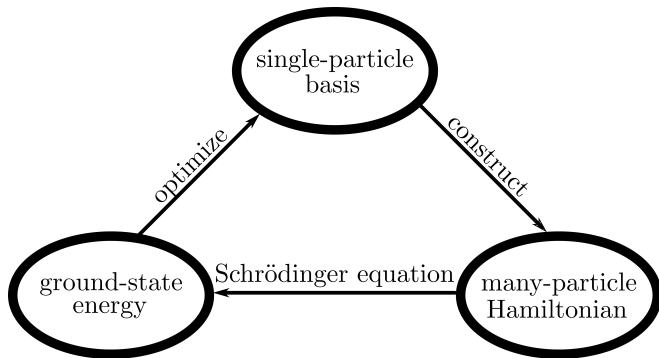
## $\text{LaH}_{10 \pm x}$



$L_{\text{theory}}$ : Hanyu Liu et al., PNAS **114**, 27 (2017)

$R_{\text{experiment}}$ : M. Somayazulu et al.,  
arXiv:1808.07695 (2018)

# Exact Diagonalization **Ab Initio** (EDABI)++

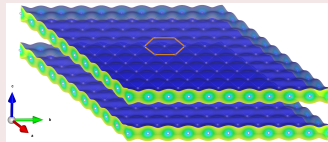
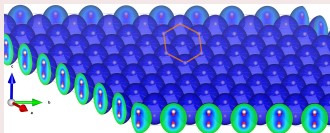


## Sources

- ♠ J. Spátek et al., Phys. Rev. B 61, 15676 (2000); ♣ APK et al., Eur. Phys. J. B 86, 252 (2013);
  - ♦ A. Biborski, APK, J. Spátek, Comput. Phys. Commun. 197, 7 (2015);
  - ♥ A. Biborski, APK, J. Spátek, Phys. Rev. B 98, 085112 (2018).
- Coming soon: EDABI for  $f$  electrons..

# Triangular lattice

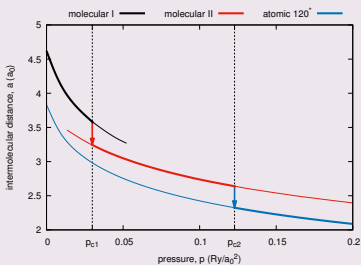
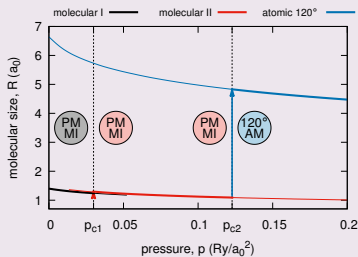
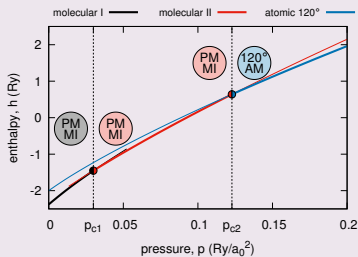
## Two-dimensional crystal



- periodic boundary conditions in  $xy$  plane;
- Lanczos algorithm for the diagonalization core of 6 and 8 atoms (to comply with proper Néel  $120^\circ$  and  $90^\circ$  phases);
- wavefunction constructed from 10 classes of nodes

$$\begin{aligned}
 \mathcal{H} = & \sum_{i\sigma} \epsilon_i \hat{n}_{i\sigma} + \sum_{i \neq j \sigma} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} && \hookrightarrow \text{hoppings } t_{ij} \text{ up to } 10^{\text{th}} \text{ neighbor;} \\
 & + \sum_i U_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \sum_{i \neq j} K_{ij} \hat{n}_i \hat{n}_j && \hookrightarrow \text{Coulomb repulsion } K_{ij} \text{ up to } 10^{\text{th}} \text{ neighbor;} \\
 & - \sum_{i \neq j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \sum_{i \neq j} J_{ij} \hat{n}_i \hat{n}_j && \hookrightarrow \text{ferromagnetic exchange } J_{ij} \\
 & + \sum_{i \neq j} J_{ij} \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\downarrow}^\dagger \hat{c}_{j\downarrow} \hat{c}_{j\uparrow} && \text{up to } 3^{\text{rd}} \text{ neighbor;}
 \end{aligned}$$

## 2D enthalpy and lattice parameters



Question:

What is the quantum equivalent of  $R_{\text{eff}} \rightarrow \infty$ ?

# Atomcity

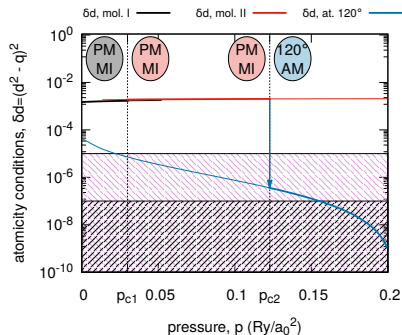
## Classically

Interplanar distance  $R_{eff} \rightarrow \infty \leftarrow$  **Not necessarily in the quantum realm!**  
(van-der-Waals-like behavior)

## Independence of classical probability

$$\delta d \equiv \left( P \begin{pmatrix} * \\ \uparrow\downarrow \end{pmatrix} P \begin{pmatrix} \uparrow\downarrow \\ * \end{pmatrix} - P \begin{pmatrix} \uparrow\downarrow \\ \uparrow\downarrow \end{pmatrix} \right)^2$$

$$\equiv \left( \langle \Phi_0 | \hat{n}_{1\uparrow} \hat{n}_{1\downarrow} | \Phi_0 \rangle \langle \Phi_0 | \hat{n}_{2\uparrow} \hat{n}_{2\downarrow} | \Phi_0 \rangle - \langle \Phi_0 | \hat{n}_{1\uparrow} \hat{n}_{1\downarrow} \hat{n}_{2\uparrow} \hat{n}_{2\downarrow} | \Phi_0 \rangle \right)^2$$





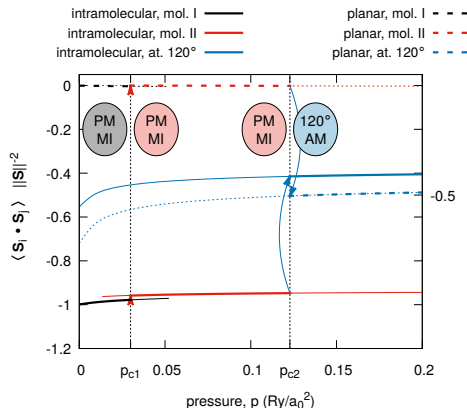
# Magnetic order

## FM vs. AFM exchange

$J_{\text{FM}}$ , Hund-like  $\ll J_{\text{AFM}}$ , kinetic  
Required for the ambient pressure stability of the atomic phase!

## Spin correlation

- 1 Molecular phases: molecular near spin-singlet  $H_2$
- 2 Atomic phase:  $120^\circ$  Néel order



## Total spin

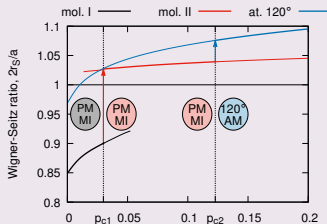
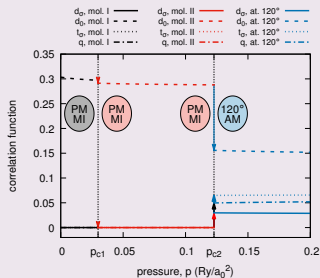
	mol. I $\rightarrow$ II		mol. II $\rightarrow$ atomic	
$\ \mathbf{S}\ _{\text{molecule}}$	0.10	0.14	0.16	0.54
$\ \mathbf{S}\ _{\text{triangle}}$	0.86	0.87	0.86	0.077

$$\|\mathbf{S}\|_{\text{molecule}} \equiv \|\mathbf{S}(x_2\mathbf{D}, -\frac{R}{2}) + \mathbf{S}_2(x_2\mathbf{D}, \frac{R}{2})\|$$

$$\|\mathbf{S}\|_{\text{triangle}} \equiv \|\mathbf{S}(x_2\mathbf{D}, \frac{R}{2}) + \mathbf{S}(x_2\mathbf{D} + \mathbf{e}_1, \frac{R}{2}) + \mathbf{S}(x_2\mathbf{D} + \mathbf{e}_2, \frac{R}{2})\|$$

# Two-step metallization

## Metallicity of atomic phase



$$q \equiv P \begin{pmatrix} \uparrow\downarrow \\ \uparrow\downarrow \end{pmatrix} \quad d_0 \equiv P \begin{pmatrix} \uparrow & \\ & \downarrow \end{pmatrix}$$

$$t_{\uparrow} \equiv P \begin{pmatrix} \uparrow & \\ & \uparrow\downarrow \end{pmatrix} \quad d_{\uparrow} \equiv P \begin{pmatrix} \uparrow & \\ \uparrow & \end{pmatrix}$$

$$t_{\downarrow} \equiv P \begin{pmatrix} & \downarrow \\ \uparrow\downarrow & \end{pmatrix} \quad d_{\downarrow} \equiv P \begin{pmatrix} & \downarrow \\ & \downarrow \end{pmatrix}$$

(top): occupancy correlation functions

(bottom): Wigner-Seitz metallicity condition

$$r_S \equiv \left( \frac{3}{4\pi n} \right)^{1/3}$$

# Band structure

## Bare bands

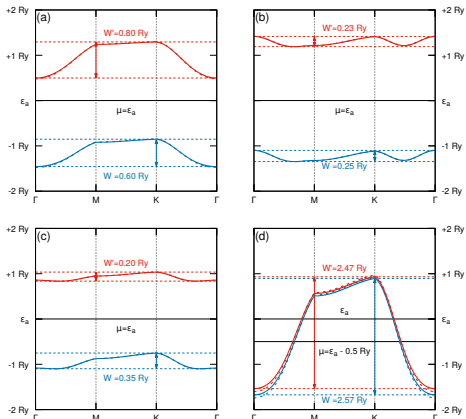
- easily calculable
- depend only on  $\mathcal{H}_{\text{free}}$

## Correlated bands

- full  $\mathcal{H}$  dependence
- no generic method

## Bare bands with a correlator

- calculable
- local interaction
- ⊗ correlator physics



# Possibility of superconducting state

## Wigner-Seitz radii

$$r_s = r_s(V)$$

- vol. of an electron in ph. I & II:

$$V_e = \frac{V_{\text{mol}}}{2} \equiv \frac{1}{2} a^2 (R + \frac{2}{\zeta}),$$

- vol. of an **atom** in atomic phase:

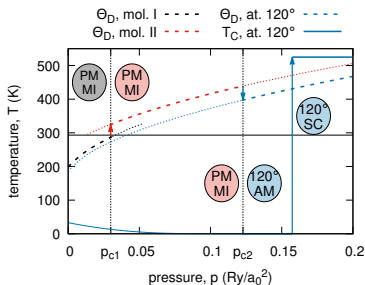
$$V_e = a^2 \frac{2}{\zeta},$$

source	method	$r_s$ ( $a_0$ )
J. McMinis et al. (arXiv:1309.7051)	DMC	2.27
G. Mazzola et al. (Nat. Commun. 5, 3487 (2014))	DMC	1.28
J.-L. Luet al. (Phys. Rev. B 66, 035102 (2002))	LSDA	2.78
J.-L. Luet al. (Phys. Rev. B 66, 035102 (2002))	GGA	2.50
B. I. Min et al. (Phys. Rev. B 33, 324 (1986))	LMTO-LSDA	2.85
A. Svane et al. (Solid State Commun. 76, 851 (1990))	SIC-LSDA	2.45
B. G. Pfommer et al. (Phys. Rev. B 58, 12680 (1998))	GGA-PW91	2.5
APK, AB, JS (2018)	EDABI	1.265
R. P. Dias et al. (Science: 10.1126/science.aal1579 (2017))	experiment	1.255 - 1.34

## McMillana formula

$T_C$  depends on

- $\Theta_D$  (from phonon spectra)
  - ⊗ always a soft mode
  - ⊥ to the plane
- $\alpha \approx 1.0$
- $\lambda^2 \approx 0.166 r_s$ 
  - ⊗ tolerable (?) approx.



# Conclusions

## Physics of hydrogen planes

- concomitant atomization & metallization;
- long-range interactions ( $\sim ||R||^{-P}$ )
- London-like interactions in insulating molecular phases;
- benchmark for infinite-system quantum

chemistry (EDABl + );

## Hydrogen-induced superconductivity

- medianly correlated system (playground for a physicist)
- (most probably)[\[citation needed\]](#) anharmonic phonons;
- (but maybe)[\[citation needed\]](#) correlation driven;
- extreme pressure (chemical?);
- record high  $T_C$ ;

Thank you for your attention

